INTRINSIC STRESSES IN CONCRETE DURING FREEZING

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Results are shown of a study concerning the intrinsic strains and stresses in frozen concrete conglomerate, also the relation is shown between unit stresses and frost resistance.

1. The effect of temperature on capillary-porous materials is accompanied by several other effects which have to do with the presence of liquid or gas in the voids [1]. The freezing of concrete saturated with water or saline solutions causes damage to concrete and reinforced-concrete structures and buildings. Although corrosion during freezing is obviously related to the phase transformation from water to ice inside the pores of concrete, it is quite difficult to explain the sequence of events leading to breakdown. Hundreds of studies have been published concerning the frost resistance of concrete. Attempts to explain the causes of breakdown were first made decades ago [2, 3] and continue to this very day [4, 5]. Various authors recognize different factors as the main cause of breakdown, namely: the hydraulic pressure of water displaced by ice, the crystallization pressure of ice, the particular mode of crystallization pressure buildup in connection with the segregation of ice crystals in the micropores of a capillary-porous hard cement structure, the formation of "ice lenses," the osmotic pressure, the migration of water through the solid porous medium toward the contact surface with ice, the higher viscosity of water displaced by ice, due to the drop in temperature and the rise in the concentration of salts dissolved in water during freezing, adsorption effects, etc. This long list clearly points to the complexity of the phenomenon and to the lack of a definitive causal explanation for it.

An external manifestation of concrete breakdown due to frost is the expansion of the structure caused by alternate freezing and thawing, accompanied by the appearance of microcracks and macrocracks on the average uniformly throughout the volume of material. Such a mode of concrete breakdown is obviously related to intrinsic (distinct from thermoelastic) structural tensile stress produced during freezing. No thorough analysis of causes and conditions could be found in the technical literature. In this study the author considers the frost resistance of concrete in the phenomenological interpretation.

2. Concrete may be treated as a two-component material consisting of an elastoplastic matrix (cement) with grains of a filler (as, for example, unslaked clinker grains) bonded to the matrix along the contact surface. The cement matrix is a capillary-porous body, while clinker with a fine filler (sand) and a coarse filler (rubble or gravel) in plain concrete constitutes compact stone. This structural diversity ensures a diversity of deformation characteristics among the ingredients of concrete conglomerate, particularly of their intrinsic strains.

When concrete saturated with water freezes, the matrix expands [6] as a consequence of the anomalous properties of water within the temperature range from −10 to −45°C, while the filler contracts regularly.

Thus, the thermal expansivity α of the capillary-porous ingredients of concrete at below-zero temperatures is, as a rule, variable. Expansion of concrete during freezing is precisely the result of its prior saturation with water (here and later on expansion will refer to concrete cooled through the said temperature range). Dry concrete does not expand and almost does not break down during freezing. It has thus been recognized that the amount of expansion of wet concrete during freezing characterizes its frost resistance.

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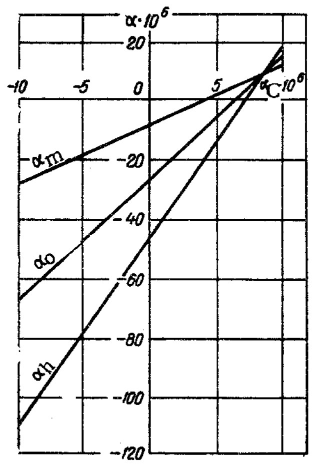

Fig. 1

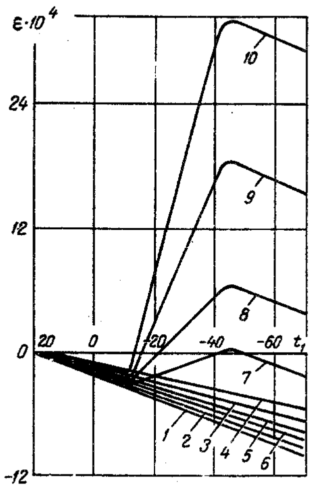

Fig. 2

Fig. 1. Thermal expansivities α_m, α_o, and α_h as functions of α_C.

Fig. 2. Unit strain ε as a function of the instantaneous temperature t_i (°C), for concrete and reinforced-concrete ingredients: 1) structural steel ($\alpha = 12 \cdot 10^{-6}$); 2) sandstone and quartzite as filler ($\alpha = 11 \cdot 10^{-6}$); 3) granite and limestone as filler ($\alpha = 8 \cdot 10^{-6}$); 4) basalt and gabbro as filler ($\alpha = 6 \cdot 10^{-6}$); 5) clinker ($\alpha = 9 \cdot 10^{-6}$); 6) dry concrete ($\alpha = 10 \cdot 10^{-6}$ 1/°C); saturated with water: 7) heavy concrete; 8) cement—sand mortar; 9) hard cement; 10) hydrated mass.

Structure levels of concrete under intrinsic strains are listed in Table 1, according to which a concrete structure may be regarded as one of the "conglomerate within a conglomerate" type. Deformations of this structure at level I must necessarily occur at lower levels. Moreover, the magnitude of the unit strain must increase during transition from level I to level III, owing to the continuity of transitions from level to level and owing to the existence at each level of compact and nominally (relative to the matrix) nondeforming ingredients. We note that the discrete values regarded here as characterizing the various levels represent somewhat arbitrarily selected sections across the conglomerate material.

Regarding concrete as a continuous solid three-dimensional medium and assuming the strain in a conglomerate to be an additive function of the strains in its ingredients, we may use here the "mixture rule" and write

$$\varepsilon_{con} = \frac{\varepsilon_1 v_1 + \varepsilon_2 v_2}{v_1 + v_2}. \tag{1}$$

In this equation ε can obviously be replaced by α for both concrete and its ingredients, according to the relation $\varepsilon = \alpha \Delta t$. Here for the capillary-porous ingredients ε is defined as the total strain in each, equal to the sum of the thermal strain in the crystalline skeleton and the bulk effect of phase transformation inside the pores. Since the change in volume of the intraporous phase is, as a rule, only partly transmitted to the structure, hence the definition or the calculation of ε must be based on a quantity which can be determined experimentally.

The absence of elasticity constants in Eq. (1) derives from an assumption that the deformation energy in the matrix is spent essentially on free expansion of the structure and causes only minor internal stresses in the conglomerate. This deformation anomaly may be attributed to the bond, characteristic of concrete, between rigid and elastic inclusions through the cement interlayer with elastoplastic properties.

Considering that $v_1 + v_2 = 1$ and appropriately replacing ε by α, we obtain from (1)

$$\alpha_{con} = v_1 (\alpha_1 - \alpha_2) + \alpha_2. \tag{2}$$

TABLE 1. Structure Levels of Strained Concrete

Level number	System	Ingredients	
		compact	capillary-porous
I	Concrete	Coarse filler	Cement–sand mortar
II	Cement–sand mortar	Fine filler	Hard cement
III	Hard cement	Clinker grains	Hydrated mass

Therefore, $\alpha_{con} = \alpha_2$ when $v_1 = 0$ and $\alpha_{con} = \alpha_1$ when $v_1 = 1$. According to (2), α_{con} is proportional to the volume content of filler in the mixture. The test data in [7] and their interpretation confirm the validity of Eq. (2). (It must be pointed out that in [7] the authors have not derived any analytical formulas to generalize their data; they have focused their attention on other aspects of the problem, instead, without considering those which we are discussing here.)

Knowing α_C, α_{cf}, v_{cf}, and v_m (the composition of concrete), one can find α_m from formula (1):

$$\alpha_m = \frac{\alpha_C - \alpha_{cf} v_{cf}}{v_m} \tag{3}$$

and then, with aid of analogous formulas, also α_o and α_h.

Assuming some typical composition of concrete, characterized by the ratio of volumes $v_h : v_{cl} : v_{ff} : v_{cf} = 0.16 : 0.09 : 0.25 : 0.5$ and $\alpha_{cf} = \alpha_{ff} = \alpha_{cl} = 9 \cdot 10^{-6}$ (°C)$^{-1}$, one can calculate α_m, α_o, and α_h for various values of α_C. The results of such a calculation are shown in Fig. 1, where the values of α_C based on the test data in [8] characterize the strain due to freezing of various concrete structures with various moisture content levels. A negative value of α indicates that the given ingredient expands during freezing. Noteworthy in Fig. 1 are the unusually high values of α_o and α_h, several times higher than their values used in practice. Without going into detail, we will refer here to test data confirming directly [8] or indirectly that the various thermal expansivities calculated according to formula (3) fall within conventional limits and correspond to their true values for the respective concrete ingredients during freezing. The different values of thermal strain in reinforced concrete, concrete, and the ingredients of both have been plotted in Fig. 2 from test data, handbook data, and calculations based on $\alpha_C = -10 \cdot 10^{-6}$ (°C)$^{-1}$. The consequences of different strains in steel and in concrete respectively had been discussed earlier in [6, 8].

3. Having determined the intrinsic strains at various structure levels, one can now calculate the intrinsic stresses produced in concrete during freezing. For this is needed a mathematical model. Of interest are an experimental model [9] and a theoretical model [10] of the concrete structure. The model in [10] is alas unsuitable for calculating the thermal stresses, because the filler has been assumed there nondeformable. We will, therefore, use the model shown in Fig. 3 as our mathematical model of the concrete structure where the concrete conglomerate consists of filler material confined to a spherical volume inside a spherical shell of cement [11, 12]. This shell contains cement—sand mortar when around coarse filler grains, it contains hard cement when around fine filler grains, and it contains hydrated mass when around clinker grains.

We will assume that such a structure element (Fig. 3) is located in an isothermal field without temperature gradients across it. At the temperature t_0 (the conglomeration temperature, tentatively) there are no stresses in this element. In practice no stresses are induced during cooling as long as $\alpha_1 = \alpha_2$. After the temperature t_{be} (beginning of expansion ≈ −10°C) has been reached, $\alpha_1 \neq \alpha_2$ and in the element there appear intrinsic structural stresses. These stresses will decrease, as the temperature drops below t_{ee} (end of expansion ≈ −45°C).

The maximum tensile stresses appear in the shell along its contact with the inner core. These are tangential (circumferential) stresses which can cause a rupture of the shell, and radial stresses which detach it from the core (Fig. 3). The stresses inside the core are less dangerous, of course, if only because the ultimate strength of the filler materials is much higher than that of cement.

Let us express the maximum stresses in a shell on the basis of the solution to the Lamé problem of an elastic sphere under internal and external pressure [13]:

TABLE 2. Stresses in a Shell of a Structure Element, as Functions of the Diameter 2a (mm) of the Filler Core in Various Systems

Kind of stress	Clinker grains and hydrated mass			Fine filler and hard cement			Coarse filler and cement–sand mortar		
	0,03	0,09	0,15	0,15	1,0	4,0	5,0	20,0	40,0
Radial σ_r'	11,9	7,2	4,6	6,5	1,25	0,2	5,95	3,7	2,3
Tangential σ_t	—8,95	—11,4	—12,7	—9,7	—12,5	—13,4	—3,6	—5,3	—6,0

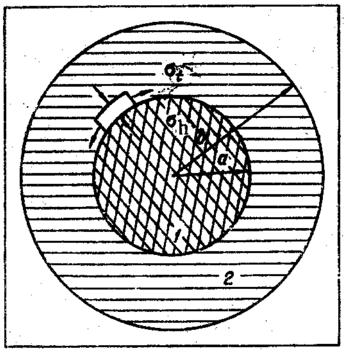

Fig. 3. Structure element (mathematical model): 1) spherical core; 2) spherical shell.

$$\sigma_r = \frac{-\Delta\varepsilon\left(1-\frac{a^3}{b^3}\right)}{\frac{1}{2E_2}\left[\left(2\frac{a^3}{b^3}+1\right)-\mu_2\left(4\frac{a^3}{b^3}-1\right)\right]+\frac{1}{E_1}\left(1-\frac{a^3}{b^3}\right)(1-2\mu_1)}, \quad (4)$$

$$\sigma_t = \frac{\Delta\varepsilon\left(2\frac{a^3}{b^3}+1\right)}{\frac{1}{E_2}\left[\left(2\frac{a^3}{b^3}+1\right)-\mu_2\left(4\frac{a^3}{b^3}-1\right)\right]+\frac{2}{E_1}\left(1-\frac{a^3}{b^3}\right)(1-2\mu_1)}. \quad (5)$$

In our case here

$$\Delta\varepsilon = (\alpha_1 - \alpha_2)\,\Delta t, \quad (6)$$

where $\Delta t = t_i - t_{be}$ when $t_{be} > t_i > t_{ee}$ and $t_i = t_{ee}$ when $t_i < t_{ee}$.

We will now calculate the stresses produced in freezing concrete under various structural conditions. For this we use the values of α_m, α_o, and α_h for $\alpha_C = -5 \cdot 10^{-6}$ (°C)$^{-1}$ (Fig. 1) and typical handbook or test values for the parameters in formulas (4), (5). Let $\delta_m = 4.0$, $\delta_o = 0.25$, $\delta_h = 0.015$ mm; $E_m = 250 \cdot 10^3$, $E_0 = 180 \cdot 10^3$, $E_h = 130 \cdot 10^3$, $E_{cf} = 500 \cdot 10^3$, $E_{ff} = 600 \cdot 10^3$, $E_{cl} = 560 \cdot 10^3$ kg/cm^2; $\mu_m = 0.22$, $\mu_0 = 0.25$, $\mu_h = 0.28$, $\mu_{cf} = \mu_{ff} = \mu_{cl} = 0.17$. The results of calculations are listed in Table 2, where the values of σ' for the cooling half-cycle are given in kg/cm^2. An examination of expressions (4)-(6) shows that, depending on the sign of the parameters here, either the radial or the tangential stresses in the shell of a structure element can be tensile. We note that there is experimental evidence available indicating two ways in which freezing concrete can break down: through cracks running along the contact between filler and cement (i.e., as a result of radial stresses) and through cracks running perpendicularly to the filler surface (i.e., as a result of tangential stresses) [14, 15].

Considering the tentativeness of the mathematical model, the qualitative relations revealed in our analysis and shown in Table 2 are sufficiently reliable. Indeed, the model has not been sufficiently refined to take into account the effect of adjacent structure elements in a conglomerate and neither have the plastic properties of the ingredients which contain hydrated mass been accounted for. Calculations show that accounting for the creep of concrete, which increases during freezing [16], results in several times lower stresses calculated according to the model. Noting the high values of stresses in Table 2, we must consider that they had been obtained on the basis of $\alpha_C = -5 \cdot 10^6$ (°C)$^{-1}$, i.e., on the basis of concrete nonresistant to frost. According to (6), $\Delta\varepsilon$ and thus also σ_r and σ_t are maximum when α_1 and α_2 have opposite signs. As α_2 increases to become positive, the stresses decrease rather fast.

4. So far, analyzing the concrete structure as one of the "conglomerate inside a conglomerate" type, we have considered that (in terms of the mathematical model) the cement—sand mortar forming a shell around coarse filler grains does itself contain many sand grains with hard cement shells, while hard cement, in turn, consists of many clinker grains with shells of hydrated mass around. Stresses appearing in the matrix, i.e., in the shells of structure elements at each structure level are superposed on one another. The superposition principle yields, with the conglomerate regarded as a continuous medium,

$$\sigma_\Sigma = \sigma_I + \sigma_{II} + \sigma_{III} \quad (7)$$

Naturally, the life of freezing concrete conglomerate is determined not by the absolute level of stresses but by what fraction of the strength of the material they represent. The criterial ratio characterizing the strength of concrete conglomerate under intrinsic stresses will be defined as follows

$$\frac{\sigma_\Sigma}{R} < 1, \qquad (8)$$

where, to the first approximation, $R = R_h$. On the basis of (8), we can now propose a set of a strength criteria for the frost resistance of concrete. The more the fraction σ_Σ/R_e decreases below unity, the lower is the probability of cracks appearing in the material due to cold effects, the higher is the stability of the material and the longer is its life. The more this fraction increases above unity, on the other hand, the faster does the life of concrete become shorter. Phenomena related to durability and to fatigue strength under cyclic loading do, apparently, lower the criterial limit defined in (8). Thus, the problem of selecting frost resistant concrete while satisfying the other requirements in terms of strength, set, creep, etc. as well as in terms of technicoeconomic limitations reduces to a minimization of the criterial fraction (8).

5. In order to clarify the physical aspects of the phenomenon which occurs in concrete during freezing, it is necessary to represent the statistical lot of data on local stresses in a structure at various levels and their corresponding spectral distribution. According to rough estimates, 1 dm^3 of concrete contains approximately $1 \cdot 10^5$ sand grains and $1 \cdot 10^8$ clinker grains, i.e., the number of compact ingredients of different structure levels in concrete can be characterized by the approximate ratio $n_I : n_{II} : n_{III} = 1 : 1.5 \cdot 10^3 : 1.5 \cdot 10^6$, which sufficiently justifies the statistical approach.

Important also are notions concerning the specifics of crack buildup in a porous structure, inasmuch as the formation of cracks is related to internal stresses. In concrete nonresistant to frost, high stresses in the cement, especially under repeated and cyclic loads, cause cracks. Cracks do, naturally, lower the strength of concrete and, while increasing its porosity and constituting stress centers, they lower its resistance to long cyclic frost effects. On the other hand, such cracks do also drastically reduce thermal stresses in a structure. While cracks due to external loads result in higher stresses and accelerated breakdown, cracks due to intrinsic stresses, on the contrary, reduce or even remove the latter. Initially (during the first few freezing cycles or at higher below-zero temperatures) there appear relatively few cracks in the structure. These cracks relieve stresses, and new repeated loads are necessary to produce critical stresses resulting in new cracks due to a supplementary saturation of concrete with water or due to a redistribution of the water [6]. This, one must assume, essentially explains why a concrete structure under high unit stresses does not disintegrate completely during the first few frost cycles.

Under intrinsic stresses, therefore, the presence of critical stresses at individual locations does not determine the breakdown of an entire structure, such a breakdown being the cumulative result of all defects. The rate of defect accumulation depends on many variable structural and external factors, making it extremely difficult to calculate the frost resistance and thus justifying a merely comparative estimate. Such an estimate may be based on Eqs. (4), (5), and (8).

6. On the basis of the preceding analysis, the frost resistance of concrete can be phenomenologically characterized as follows. The frost resistance is determined by the relative magnitude of intrinsic tensile stresses appearing in a concrete structure during freezing and thawing. Breakdown stresses during temperature variations appear in the conglomerate as a result of the difference between the respective thermophysical properties of its ingredients. The difference between the thermal strains in the ingredients of frozen concrete saturated with water is determined mainly by the presence here of ingredients anomalously expanding during freezing.

The intensity of the state of stress is determined by the magnitude of the total stresses resulting from a superposition of stresses at each structure level.

According to the concepts developed here, the frost resistance of concrete is affected by all factors which determine the strength of concrete before freezing as well as by the magnitude of intrinsic stresses due to freezing and thawing. The vast number of these factors and the difficulty in accounting for their various combinations determine the complexity of an experimental study dealing with this matter.

An analysis of published data on the frost resistance of concrete has shown and an experiment has verified that, on the basis of the concepts presented here, it is possible to interpret with a single criterion the large multitude of test data on this subject. A close quantitative agreement between theoretical test values has been found in several cases. This applies, for instance, to the effects of concrete "swelling," of the filler material, etc. on the frost resistance.

The results can obviously be used for solving technological problems as well as problems of durability of concrete at temperatures and under corrosive or other conditions when intrinsic strains and structural stresses are produced.

NOTATION

ε	is the unit strain;
$\Delta\varepsilon$	is the difference of unit strains;
V	is the volume concentration;
α	is the thermal expansivity;
t_i	is the instantaneous temperature;
t_0	is the initial temperature;
t_{be}	is the temperature at which expansion during cooling begins;
t_{ee}	is the temperature at which expansion during cooling ends;
σ	is a stress;
σ'	is a unit stress;
a	is the core radius;
b	is the outside radius of the shell;
$\delta = b - a$;	
E	is the modulus of elasticity;
μ	is the Poisson ratio;
R	is the strength (local).

Subscripts

1	refers to the filler (the core in the model);
2	refers to the cement (the shell in the model);
con	refers to conglomerate;
C	refers to concrete;
m	refers to mortar;
o	refers to hard cement;
h	refers to hydrated mass;
cf	refers to coarse filler;
ff	refers to fine filler;
cl	refers to clinker;
r	denotes radial (stress);
t	denotes tangential (stress);
Σ	denotes total;
I, II, III	refer to the first, the second, and the third structure levels respectively;
T	denotes tensile (stress).

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